

# Design principles for broadband AR coatings

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## ABSTRACT

It is shown that most of the known AR coatings that start with the famous quarter-half-quarter coating including the multi-cycle designs display simply alternative solutions to replace unavailable refractive indices of a standard solution. On the basis of this viewpoint some design principles are presented that help to derive an approach to solve a given AR problem. Principal objective of the presented approach is the application of previously defined terms 'equivalent substrate index' and 'equivalent stack index' that characterise a quarter-wave optical thickness multilayer system. It is shown that the presented design principles are also suitable if, instead of a thin-film system, a modification of the interface is used to reduce the surface reflection.

**Keywords:** AR coatings, optical thin-films, design

## 1 INTRODUCTION

Any optical device includes at least one optical element with a refractive index different from the refractive index of the ambient medium. This difference in refractive indices in the direction of the beam propagation produces a reflection that is unwanted in most of the cases. This problem is called the 'anti-reflection' problem or 'AR problem' and has been discussed and solved, more or less, throughout the times of modern optics since the middle of the last century.<sup>1-7</sup> The simplest solution to this problem is the deposition of a thin film onto the substrate provided that the refractive index of this thin film is lower than the refractive index of the substrate. The first practical application of such a single-layer AR coating was published for the first time by Strong in 1936.<sup>8</sup> Meanwhile, numerous papers and practical solutions are known and any thin-film software enables almost anyone to solve an AR problem quickly, theoretically at least. Additionally, one has to consider another practical solution of the AR problem in form of a modification of the substrate surface, either by a specific surface structure or by air pores within the substrate material.<sup>9-12</sup> Such a surface modification, using an acid etching of the glass surface, was first published in 1904.<sup>13</sup>

The two methods differ in their technical realisation, but use the same optical principle - the interference effect of light. The thin film as well as the surface modification can be characterised by the same *effective* refractive index and also by the same physical thickness if they are used to decrease the reflection at the substrate surface within a desired wavelength range. Whatever method is applied there is a defined *transition region* between ambient medium and substrate and this transition region is characterised by its effective refractive index and its physical thickness. In this sense, it is helpful to specify the requirement for a general anti-reflective property as *AR quality* at the boundary of two media of different refractive indices. And this small alteration of the common viewpoint shall indicate the direction of our discussion. In the following, we will discuss neither any technical solution of the AR problem nor any complex formulas for its solutions, and we do not presuppose any powerful thin-film software to simulate possible solutions. In continuation of several presentations held on former conferences<sup>14-16</sup> and on recently published papers for coatings on plastic optics,<sup>17,18</sup> we present some straightforward thin-film design principles and a useful visualisation for solving the AR problem 'by reflection' (on the problem).

## 2 DESIGN PRINCIPLES

### 2.1 The AR problem and the general 'Thickness-Principle'

Without loss of generality, we assume normal incidence onto the substrate of our optical element; we do not consider the influence of the second surface of the substrate; we use constant refractive indices of all the materials and media, and we

do not have any losses in our optical arrangement. Then, our AR problem is defined by the fact that the reflectance  $R_{\text{sub}}$  at the boundary of ambient medium and substrate is unwanted and has to be reduced to a value of  $R_{\text{min}}$  within a given spectral range. This spectral range is defined by its lower and upper boundaries  $\lambda_l$  and  $\lambda_u$  which gives a bandwidth  $BW = \lambda_u/\lambda_l$  and a reference wavelength  $\lambda_0 = 2\lambda_l\lambda_u/(\lambda_l + \lambda_u)$  within this spectral range. If the BW is equal or greater than 1.5 we talk about a *broadband* AR problem.

As mentioned in the introduction, a transition region with an effective refractive index  $n_{\text{eff}}$  and a physical thickness of  $t_{\text{eff}}$  is placed in between the ambient medium and the substrate that are defined by their refractive indices  $n_0$  and  $n_s$ , respectively. There is an ideal solution with  $R(\lambda_0) = 0$  if the transition region has an optical thickness of a quarter of the reference wavelength and a refractive index equal to the square root of the product  $n_s$  times  $n_0$ , with a first practical solution fifty years ago.<sup>19</sup>

However, this single quarter-wave optical thickness (QWOT) solution or square root solution (root-solution) usually does not fulfil the requirement for a broadband AR quality. But it is also known that a broader spectral range of AR quality can be realised if the refractive index of the transition region varies continuously from the refractive index of the substrate to that of the ambient medium.<sup>20-21</sup> Such an inhomogeneous refractive index profile can also be approximated by a multilayer of homogeneous layers with different refractive indices that follow a so-called step-down profile<sup>22</sup>. This ‘broadening’ of the AR quality, however, is only possible by increasing the thickness of the transition region with respect to the QWOT solution. We can state a first but very general ‘thickness’ principle: The broader the spectral AR region shall be the thicker the AR transition region has to be. This principle is applicable to any step-down index profile, and we will qualify it in section 2.3.

## 2.2 Visualisation of the AR problem and the ‘Equivalent-Principle’

We adopt a special viewpoint on our field of interest by assuming a multilayer of different refractive indices but of equal optical thicknesses of a quarter of the reference wavelength (QWOT layers) within our transition region between the ambient medium and the substrate. Recently<sup>23</sup>, we have introduced a so-called equivalent stack index  $E$ , similar to the known Herpin Index but under renunciation of the symmetry of the layer sequence, and a so-called equivalent substrate index  $S$ , defined by

$$E^2 = \frac{M_{21}}{M_{12}} \quad \text{and} \quad S = \frac{M_{22}}{M_{11}} n_s \quad (1)(2)$$

where  $M_{11}$ ,  $M_{12}$ ,  $M_{21}$ , and  $M_{22}$  are the 2x2 elements of the characteristic matrix of the assumed multilayer using the matrix formalism.<sup>24</sup> We can act on the assumption that the limit exist if the principal and the secondary matrix elements become zero simultaneously. Using Eqs. (1) and (2) the reflectance of the multilayer can be written as

$$R = \frac{M_{11}^2 (n_0 - S)^2 + M_{12}^2 (n_0 n_s - E^2)^2}{M_{11}^2 (n_0 + S)^2 + M_{12}^2 (n_0 n_s + E^2)^2} \quad (3)$$

If there are  $p$  QWOT layers within the multilayer, we set its phase thickness to

$$\varphi_p = \frac{\pi p \lambda_0}{2 \lambda} \quad (4)$$

and assume that the remaining matrix elements in Eq. (3) are proportional to that phase thickness with

$$M_{11} \sim \cos \varphi_p \quad \text{and} \quad M_{12} \sim \sin \varphi_p.$$

On these assumptions, the reflectance acc. Eq. (3) oscillates between  $R_s$  and  $R_E$

$$R_S(\varphi_p) \approx \left( \frac{n_0 - S}{n_0 + S} \right)^2 \text{ if } \varphi_p = k\pi/2 \text{ with even } k, \text{ and } R_E(\varphi_p) \approx \left( \frac{n_0 n_S - E^2}{n_0 n_S + E^2} \right)^2 \text{ if } \varphi_p = k\pi/2 \text{ with odd } k,$$

if the phase thickness of the multilayer acc. Eq. (4) is equal to an even or an odd number of  $\pi/2$ , respectively. Because these different phase thicknesses are also met in dependence on the wavelength, our broadband AR problem is perfectly solved at the corresponding spectral positions if the equivalent stack index  $E$  is approximately equal to the square root of  $n_S \cdot n_0$  and if the equivalent substrate index  $S$  is approximately equal to the refractive index of the ambient medium.

Figure 1 shows an example with four QWOT layers on a crown glass substrate to demonstrate these conditions. This example corresponds to the 'mother' of all the AR coatings, the famous quarter-half-quarter-design given by Geffken in 1940.<sup>25</sup> It shows the known phase conditions: If the wavelength is equal to  $\lambda_0$ , the multilayer that consists of four QWOT layers has the phase value of  $2\pi$  which means the multilayer forms a so-called absentee layer of any refractive indices and the reflectance is equal to that of the substrate. However, in this case, the refractive index values of the four layers realise an equivalent substrate with an effective index of 1.0, and this means again that the reflectance is indeed zero. If the wavelength is equal to  $3/4\lambda_0$ , the phase thickness of the multilayer corresponds to a phase value  $3/2\pi$  and the equivalent substrate index shows a pole. But the  $M_{11}$  element becomes zero and the multilayer corresponds to a single layer having the equivalent stack index of 1.266. This value is approximately the required value of the root-solution acc. Eq. (2), that is, the reflectance is nearly zero.

This example provides a visualisation of what AR quality of a transition region means and can be stated as a second AR principle, the 'equivalent-principle': If the phase thickness of the transition region is equal to an integer multiple of  $\pi/2$ , the refractive indices of the transition region have to fulfil different conditions that can be defined exactly by an equivalent stack index and an equivalent substrate index. But in any case, the equivalent stack index is equal to the required effective refractive index considering the single root-solution to the special AR problem. This equivalent principle also provides the possibility to exactly determine the required refractive indices, numerically, at least.

### 2.3 Maximally flat design and the 'Phase-Broadening-Principle'

Over the years, a lot of different refractive index profiles have been introduced to realise a broadband AR quality.<sup>26,27</sup> The best known profile is the so-called maximally flat or Butterworth profile.<sup>28</sup> However, if we calculate  $S$  and  $E$  of such a profile there is big surprise in form of a constant equivalent stack index. Figure 2 shows an example of a QWOT maximally flat step-down profile on Ge with  $n_S = 4.0$ .<sup>29</sup> The equivalent stack index has exactly the value 2.0, the root-solution for a single QWOT layer. This gives the idea to state a phase-broadening principle for broadband AR quality: Whatever AR bandwidth for the transition region is required, the equivalent stack index of the region has to be equal to the required refractive index of the simple root-solution, although the phase thickness of the transition region has to be increased - but only in integer steps of  $\pi/2$ . That means, the optical thickness of the transition region has to be broadened in steps of QWOT no matter which index profile is chosen. Figure 3 shows the broadening of the spectral AR region by increasing the optical thickness and Figure 4 shows the corresponding equivalent stack and substrate indices. In this sense, the thickness-principle of section 2.1 is augmented by the phase-broadening-principle.

### 2.4 Maximally flat design and the 'Root-Principle'

The maximally flat profile can state generally for the AR quality itself: Independently of the thickness of the transition region, the reflectance show a strictly monotonic decrease down to a minimum at the reference wavelength similar to the cosine-shaped performance of the reflectance of a single QWOT transition region. Additionally, the equivalent stack index of any maximally flat profile is equal to the required refractive index for the single root-solution. These facts give the idea to see a 'root-principle' behind the relatively complex maximally flat algorithm: The refractive index given by the root-solution stands for the equivalent substrate index within the transition region between the two media. The next refractive index extension uses the root of the refractive index of the substrate (or the ambient medium) times the index of the equivalent sub-medium. And this new refractive index can be used again as an equivalent substrate index between

the substrate (or the ambient medium) and the last determined equivalent substrate to calculate a further root-solution, and so on.

If we use the index  $i$  for the number of steps of the refractive index profile, and the index  $j$  for the refractive index within the index profile, starting with  $j = 1$  as the first step on the substrate, then we have a serial refractive index  $n_{ij}$  that can be defined by the following recursive algorithm

$$n_{i(j=1)} = \sqrt{n_s n_{(i-1)(j=1)}} \quad (5a)$$

$$n_{ij} = \sqrt{n_{(i-1)(j-1)} n_{(i-1)j}} \quad (5b)$$

$$n_{i(j=i)} = \sqrt{n_{(i-1)(j=i-1)} n_0} \quad (5c)$$

This algorithm enables anybody to calculate a maximally flat solution only by applying the ‘root-principle’. We do not need the usually used R/T ratio and the expanding of that ratio into a Fourier series.<sup>30</sup>

### 2.5 Realistic index solutions and the ‘Non-Zero-Principle’

Because zero reflection can be realised only for single wavelengths and not completely over any spectral interval, it makes sense to set  $R_{\min} \neq 0$ . The root-principle acc. Eqs. (5a) to (5c) can be applied if it is modified by the substitution of the given refractive index of the ambient medium  $n_0$  by a target refractive index  $n_T$  which is determined by setting a target reflectance  $R_T = R(\lambda_0) > 0$ , and  $n_T$  can be calculated using

$$n_T = n_0 \left( \frac{1 + \sqrt{R_T}}{1 - \sqrt{R_T}} \right). \quad (6)$$

In comparison with the  $R=0$ -solution, this ‘non-zero-principle’ causes two essential effects: Firstly, all the required refractive indices within any step-down index increase and, secondly, the reflectance becomes some ripples depending of the number of steps which are acceptable, in most of the cases. Figure 5 shows the reflectance performance with  $R_T = 0$  and  $R_T > 0$  as an example of the non-zero-principle, again with the 4-layer example on Ge as of Figure 2.

Up to now we have discussed only the case of a general AR quality at the boundary of two media of different refractive indices regardless of which method is used to realise the transition region in a realistic manner or which refractive indices the step-down profiles require. All the derived principles can be applied independently of this method. If we decide in favour of the surface modification to realise an AR quality, for example on a polymer substrate, we do not need any further principles and can directly calculate the refractive indices of a maximally flat design on PMMA. Additionally, however, we have to apply the rigorous coupled wave theory<sup>31</sup> to convert the required refractive indices in a 2-dimensional cone-shaped grating, similar to the known moth-eye structure.<sup>32,33</sup> But if we decide to continue with thin-film multilayer to realise AR quality, we should take into account another principle, and we have to give the whole matter some additional considerations.

### 3 SUBSTITUTIONS OF UNSUITABLE REFRACTIVE INDICES AND THE ‘MULTI-CYCLE-PRINCIPLE’

It is not possible to apply the root-principle on low index substrates because most of refractive indices are quite low (less than 1.35) and, therefore, they are unsuitable. This fact is known from the first days of thin-film optics but to overcome this situation in principle we should change our point of view, first of all. For an AR quality on a crown glass with  $n_s = 1.5$  at 550 nm within the ambient medium air, the single root-solution with a target reflectance of 0.3 % is a QWOT layer with a refractive indices of  $n_{\text{eff}} = 1.294$ . We can substitute this unsuitable index using two or more QWOT layers in a step-up and step-down profile. But we do not use the step-down profiles mentioned above, we apply only the known admittance transformer.<sup>34</sup> The QWOT layer close to the ambient medium is in any case of a low-index material, for

example, SiO<sub>2</sub>. The maximum in our step-up profile is defined by the material with the highest available refractive index, for example, TiO<sub>2</sub>. Figure 6 shows the index profiles and Figures 7 the corresponding spectra of the step-up and step-down profiles that only substitute the unsuitable index of 1.294 of our example..

We see on the Figures, on the one hand, we get a broader bandwidth if we use more and more QWOT layers, but on the other hand, we do only simulate the unsuitable index the better the more QWOT layer we use. The layer sequence involves the lowest and the highest available refractive indices and such a solution is the best AR quality achievable with a so-called one-cycle-solution. We use the term 'cycle' from the admittance concept and define a cycle as the admittance trace from the substrate to the maximum admittance and then from the maximum to the final admittance.

However, this solution is in any case the substitution of the single root-solution. If we use the root principle to calculate two or more steps in a step-down index profile on our low index substrate we get more than one unsuitable refractive indices but each of them can be substituted by a multilayer of two and more QWOT layers. Figure 8 shows such a solutions, known as a 'multi-cycle'-design since the Berlin coating design contest 1992.<sup>35</sup> Using a multi-cycle AR designs can be seen as the application of a 'multi-cycle-principle': Each single AR root-solution can be substituted by a 'cycle' of QWOT layers and each step within a step-down index profile can be substituted by a cycle of QWOT layers, if the refractive index itself is not applicable by a material with the corresponding refractive index. Figures 9 and 10 show the multi-cycle principle in a remarkable manner: To substitute the unsuitable single root-solution, the application of a single 3-QWOT-layer cycle does not stand for a successful AR coating. However, if we use a 6-cycle-application of this 3-QWOT-layer sequence there is remarkable AR quality.

Actually, except for the given lowest and highest refractive index, all the remaining unsuitable refractive indices of such a multi-cycle QWOT-approach can be substituted conventionally by the corresponding Herpin index concept using just these given high- and low-index materials.<sup>36</sup> Finally, with respect to the real materials that show a dispersion of their refractive index versus the wavelength, a usually used refinement as an optimisation procedure with respect to the physical thickness of each of the layers has to be performed to get the correct design for an AR coating that can be manufactured with a corresponding coating technology.

#### 4 CONCLUSIONS

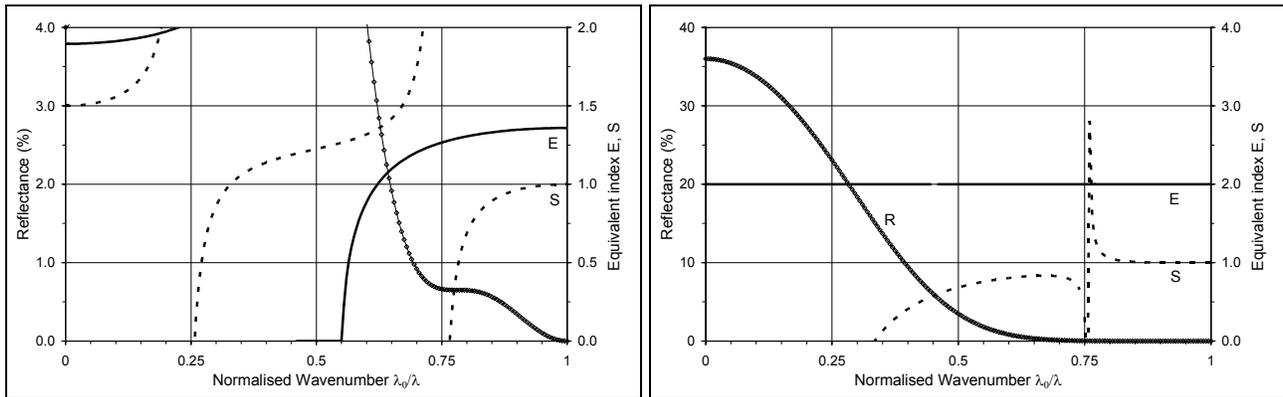
The AR problem as unwanted residual reflection at the boundary between two media of different refractive indices has been solved more or less satisfying over last 70 years. The problem itself can be seen as the need to realise a transition region between the two media with an effective refractive index as the square root of the product of the two given refractive indices and a phase thickness with an integer multiple of  $\pi/2$ . Three groups of concepts and technologies can be distinguished throughout the times up to now. The first group spans the time from the beginning of practical thin-film optics in 1936 to the middle of the 1980s. Now, looking back, we can characterise this group by the application of 'single-cycle' designs. Since the middle of the 1980s up to now we have to distinguish two other groups: One group that has overcome the AR problem by the technical realisation of nearly maximally flat designs on high index or polymer substrates, and the other group, that has further deposited multilayer coatings but now with 'multi-cycle' designs up to 20 layers and more. There are great differences between these three groups due to the used designs and technologies but it seems possible to get a homogenous consideration on all the possible solutions of the AR problem if several general principles are considered.

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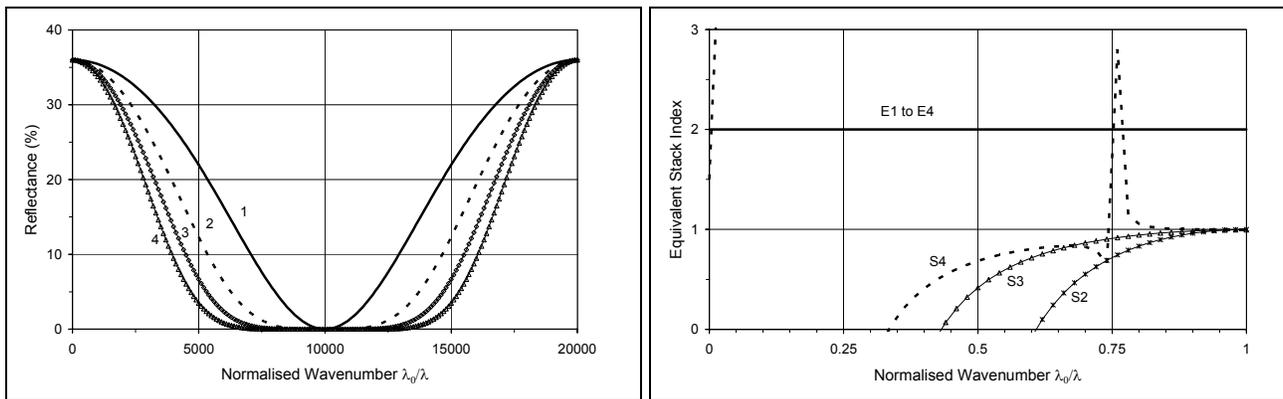
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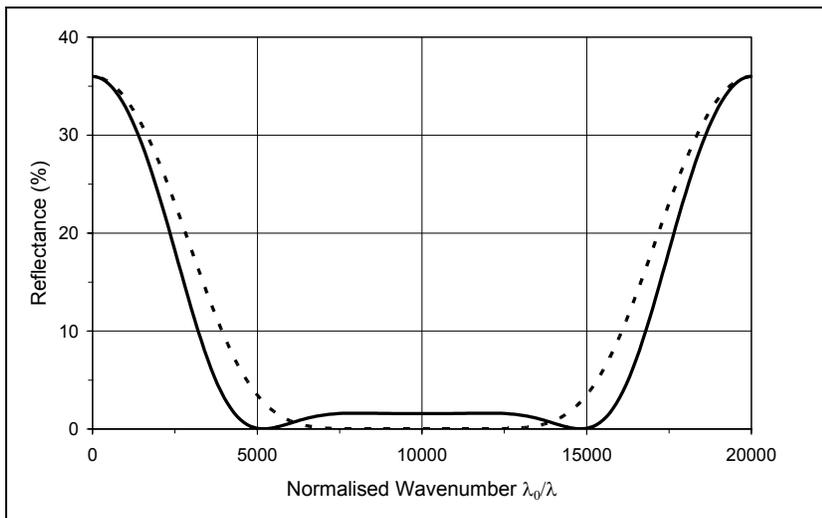
## Figures



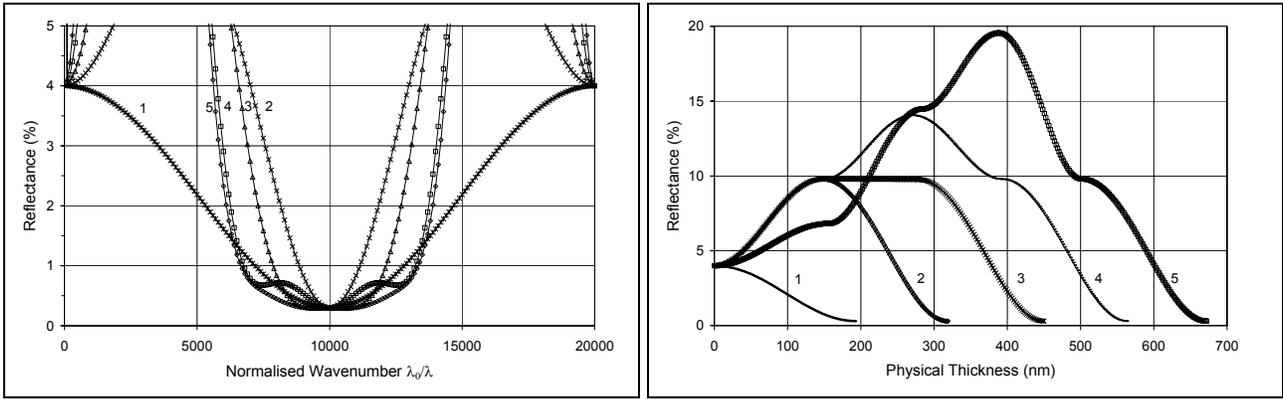
**Figures 1 and 2:** Spectral characteristic, equivalent stack index E, and equivalent substrate index S of a QWOT-design sub/M2HL/air with  $n_S = 1.5$ ,  $n_M = 1.776$ ,  $n_H = 2.24$ ,  $n_L = 1.45$ , and  $n_0 = 1.0$  (left) and sub/ABCD/air with  $n_S = 4.0$ ,  $n_A = 3.668$ ,  $n_B = 2.594$ ,  $n_C = 1.542$ ,  $n_C = 1.091$ , and  $n_0 = 1.0$  (right).



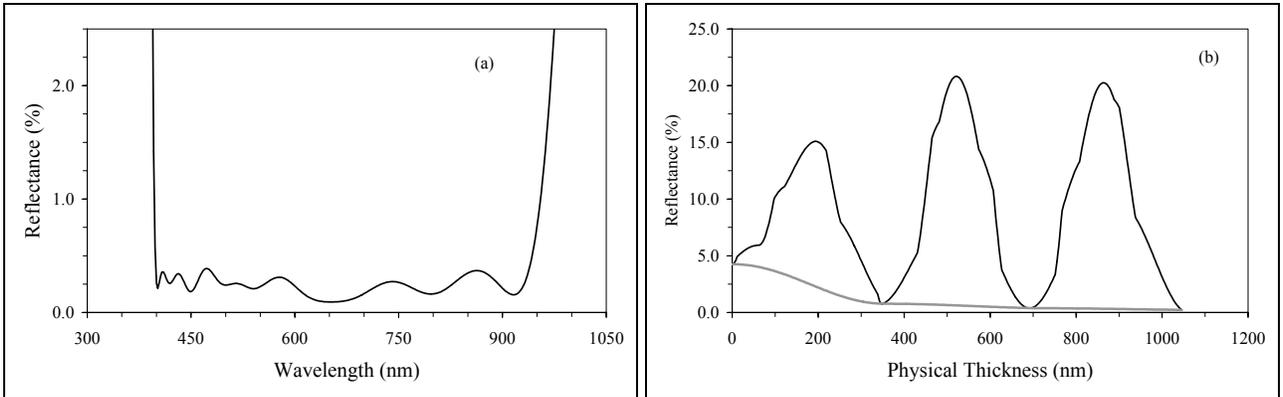
**Figures 3 and 4:** Spectral characteristics (left) and equivalent stack index E and equivalent substrate index S (right) of a 1-QWOT-layer (1), 2-QWOT-layers (2), 3-QWOT-layers (3), and 4-QWOT-layers (4) on Ge, with  $n_S = 4.0$ ,  $n_{(1)} = 2.0$ ,  $n_{(2)1} = 2.828$ ,  $n_{(2)2} = 1.414$ ,  $n_{(3)1} = 3.364$ ,  $n_{(3)2} = 2.0$ ,  $n_{(3)3} = 1.189$ ,  $n_{(4)1} = 3.668$ ,  $n_{(4)2} = 2.594$ ,  $n_{(4)3} = 1.542$ ,  $n_{(4)4} = 1.091$ ,  $n_0 = 1.0$ . For the 1-QWOT-layer  $S1 = 4.0$ .



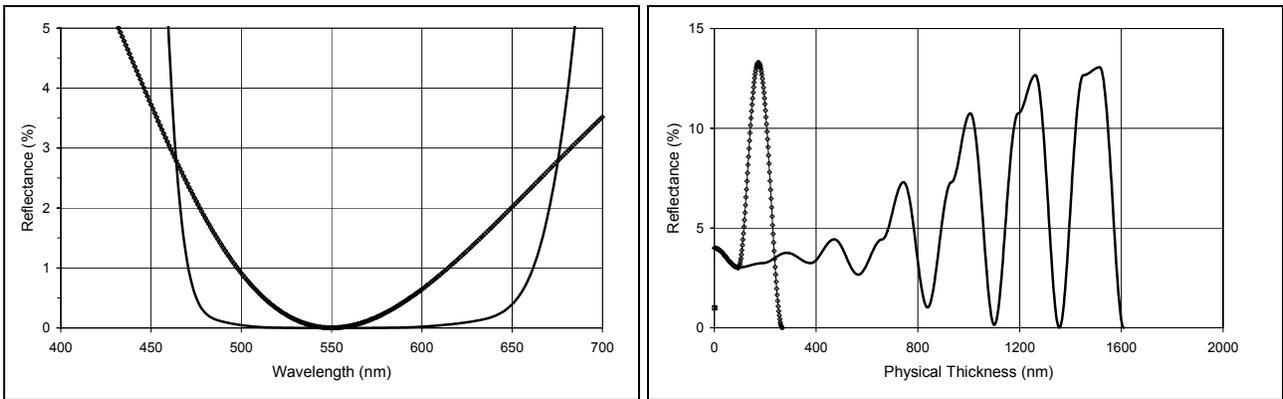
**Figure 5:** Spectral characteristic of a 4-QWOT-step-down profile on Ge with  $R(\lambda_0) = 0$  (dotted) and  $R(\lambda_0) = 1.5\%$  (thick).



**Figures 6 and 7:** Spectral characteristic (left) and thickness plot of 1-QWOT- layer (1), 2-QWOT-cycle (2), 3-QWOT-cycle (3), 4-QWOT-cycle (4), and 5-QWOT-cycle (2), with the same target reflectance of 0.3 %.



**Figure 8a and 8b:** 3-cycle AR design with 22 layers, spectral characteristic (left) and thickness plot (right),<sup>35</sup> supplemented by the reflectance of a corresponding step-down index profile.



**Figures 9 and 10:** Spectral characteristic (left) and thickness plot (right) of a 1-cycle (dotted) and a 6-cycle (thick) AR design.